MATHEMATICAL MODELING THE RISK OF GROWING SPRUCE STANDS

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SPRUCE FORESTS DECLINE IN THE BESKYDY MOUNTAINS
Reasons, Impacts, Solutions

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OBJECTIVE:

To explain the essence and assessment of risk concerning growing spruce forest stands in the presence of aggregated forest management risk.

CONTENT:

- Introduction
- Definitions of basic concepts
- Paradigm of risk and its shifting
- Description of hazard
- Measurement of risk
- Economic analysis of risk
- Conclusions
- Institutional framework for accepting the risk in forestry

INTRODUCTION

Spruce is the most endangered tree species grown in the territory of Slovakia.

A QUESTION:

Can we know the risks we face, now or in the future?

AN ANSWER:

No, we cannot; but we must act as we do.

(Douglas & Wildawsky, 1983)

Definition of hazard:

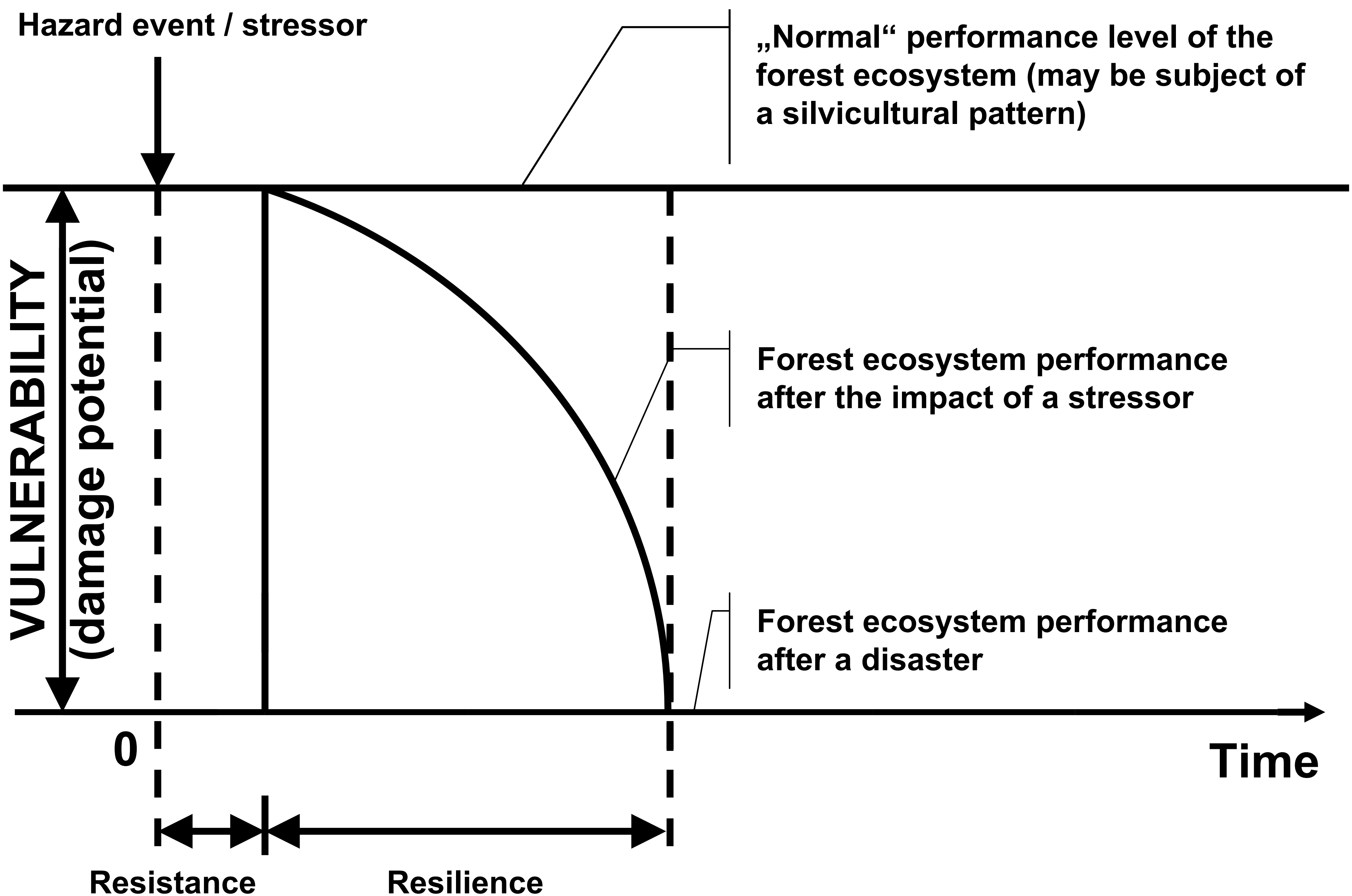
HAZARD is a thread to humans or what they value.

Alternatively:

HAZARD is a situation that in particular circumstances could lead to harm (human being) or damage (loss of inherent quality) suffered by a biological or physical entity.

We distinguish different kinds of hazard (bolt striking, wind-throw, insects, fire, diseases, earthquake …)

Definition of vulnerability:



Visualisation of the concept of vulnerability:

Resistance + Resilience = Coping Capacity

(expressing them as the complementary quantities to the value of vulnerability)

Disaster = **when the own coping capacity (without external help) is insufficient to return the ecosystem back to the " normal " state**

The concept of the economic vulnerability of forest :

VFSV (t) = FSV (t) – SVFS (t)

VFSV (t) is the vulnerable forest stand value at age (t) and it refers to the endangered value of a forest stand

FSV (t) - the forest stand value at age (t)

SVFS (t) - the salvage value of a forest stand after its damaging or destruction at age (t)

Definition of risk:

RISK is hazard quantitatively expressed as the probability of particular natural element occurrence multiplied by the endangered value of a forest.

Risk = Probability * Vulnerability

Vulnerability = endangered value of a forest

Economic interpretation of risk concerning management of forest:

$$\textbf{Risk} = \sum_{\textit{kinds of hazard}} \textbf{Probability} * \textbf{Vulnerability}$$

Vulnerability is referred to in terms of money.

Analysed hazards:

FUNGHI, IMMISSIONS, INSECTS, WIND-THROW

A PARADIGM OF RISK AND ITS SHIFTING

Ensuring security forestry requires a paradigm shift in the concept of disaster prevention and preparedness.

Instead of starting with a focus on natural hazards and their quantification, the assessment and ranking the vulnerability of forest should serve as the starting point in defining the priorities and means of remedial interventions.

Old Paradigm	New Paradigm
Focus on natural hazards and their Quantification	Focus on the assess-ment and ranking of Vulnerability
From the „security forestry“	To the „risk forestry“

DESCRIPTION OF HAZARD

Data: Areas of spruce forests destroyed by natural elements in the territory of Central Slovakia.

Source: National Forest Inventory Database

Institute for Forest Management

Planning in Zvolen, Slovakia

Period of observation: 1993 – 2002

Area of observed forest land: 144 276 ha

MEASUREMENT OF RISK

Modelling the probabilities of forest stands destruction by the occurrence of natural elements:

Point estimate of the population proportion (*f*):

$$f = \frac{\sum_{i=1}^{k} n_i}{\sum_{i=1}^{k} N_i} \qquad \sum_{i=1}^{k} n_i = n \quad \text{and} \quad \sum_{i=1}^{k} N_i = N$$

n_i is expected destroyed area of forest within the age class (i)

N_i - expected area of forest within the age class (i)

k - number of assumed age classes

Problem:

Observed destruction probabilities related to the age of forest stands prove to be significantly different .

Solution:

Weibull probability distribution W(c,γ) described by its distribution function ***F(t)*** as proposed by KOUBA (2002) and von GADOW (2000):

$$F(t) = 1 - e^{-c.t^{\gamma}}$$

where **(*t*)** denotes the age of a forest stand.

Probability of destruction during 1 year related to the age of forest stand (*t*) is presented by **point estimates** of probabilities ***p(t)*** expressed in terms of the following relation:

$$\frac{p(t)}{k} = \frac{\Delta F(t).n}{N}$$

, i. e.

$$p(t) = k.\Delta F(t).f$$

FUNGHI AGE	AREA GROWN (ha)	AREA DESTROYED (ha)	POPULATION PROPORTION	UNIFORM AREA (ha)	DESTROYED EXPECTED (ha)
(t)	A(t)	a1(t)	f(t)	N(t)	n1(t)
10	100 564	0,0000	0,000000	96 184,29	0,0000
20	142 236	0,3630	0,000003	96 184,29	0,2455
30	120 596	4,6350	0,000038	96 184,29	3,6967
40	101 685	9,4436	0,000093	96 184,29	8,9327
50	92 353	8,0820	0,000088	96 184,29	8,4173
60	140 941	23,4453	0,000166	96 184,29	16,0001
70	156 814	43,2313	0,000276	96 184,29	26,5166
80	158 220	26,0004	0,000164	96 184,29	15,8061
90	158 583	26,1754	0,000165	96 184,29	15,8759
100	107 606	15,4539	0,000144	96 184,29	13,8136
110	54 157	9,4249	0,000174	96 184,29	16,7388
120	31 036	1,1710	0,000038	96 184,29	3,6289
130	15 429	0,0000	0,000000	96 184,29	0,0000
140	9 292	0,0000	0,000000	96 184,29	0,0000
150	53 251	0,0000	0,000000	96 184,29	0,0000
TOTAL	1 442 764	167,4257	0,001348	1 442 764,30	129,6722

f = 0,000089878

IMMISSIONS AGE	AREA GROWN (ha)	AREA DESTROYED (ha)	POPULATION PROPORTION	UNIFORM AREA (ha)	DESTROYED EXPECTED (ha)
(t)	A(t)	a2(t)	f(t)	N(t)	n2(t)
10	100 564	0,0000	0,000000	96 184,29	0,0000
20	142 236	7,9865	0,000056	96 184,29	5,4007
30	120 596	8,3109	0,000069	96 184,29	6,6286
40	101 685	8,5683	0,000084	96 184,29	8,1048
50	92 353	6,7350	0,000073	96 184,29	7,0144
60	140 941	20,9819	0,000149	96 184,29	14,3189
70	156 814	32,6695	0,000208	96 184,29	20,0384
80	158 220	18,7566	0,000119	96 184,29	11,4025
90	158 583	37,8301	0,000239	96 184,29	22,9448
100	107 606	31,7129	0,000295	96 184,29	28,3469
110	54 157	20,5264	0,000379	96 184,29	36,4552
120	31 036	22,9173	0,000738	96 184,29	71,0229
130	15 429	24,7629	0,001605	96 184,29	154,3698
140	9 292	14,3322	0,001542	96 184,29	148,3503
150	53 251	95,0511	0,001785	96 184,29	171,6849
TOTAL	1 442 764	351,1417	0,007341	1 442 764,30	706,0830

f = 0,000489396

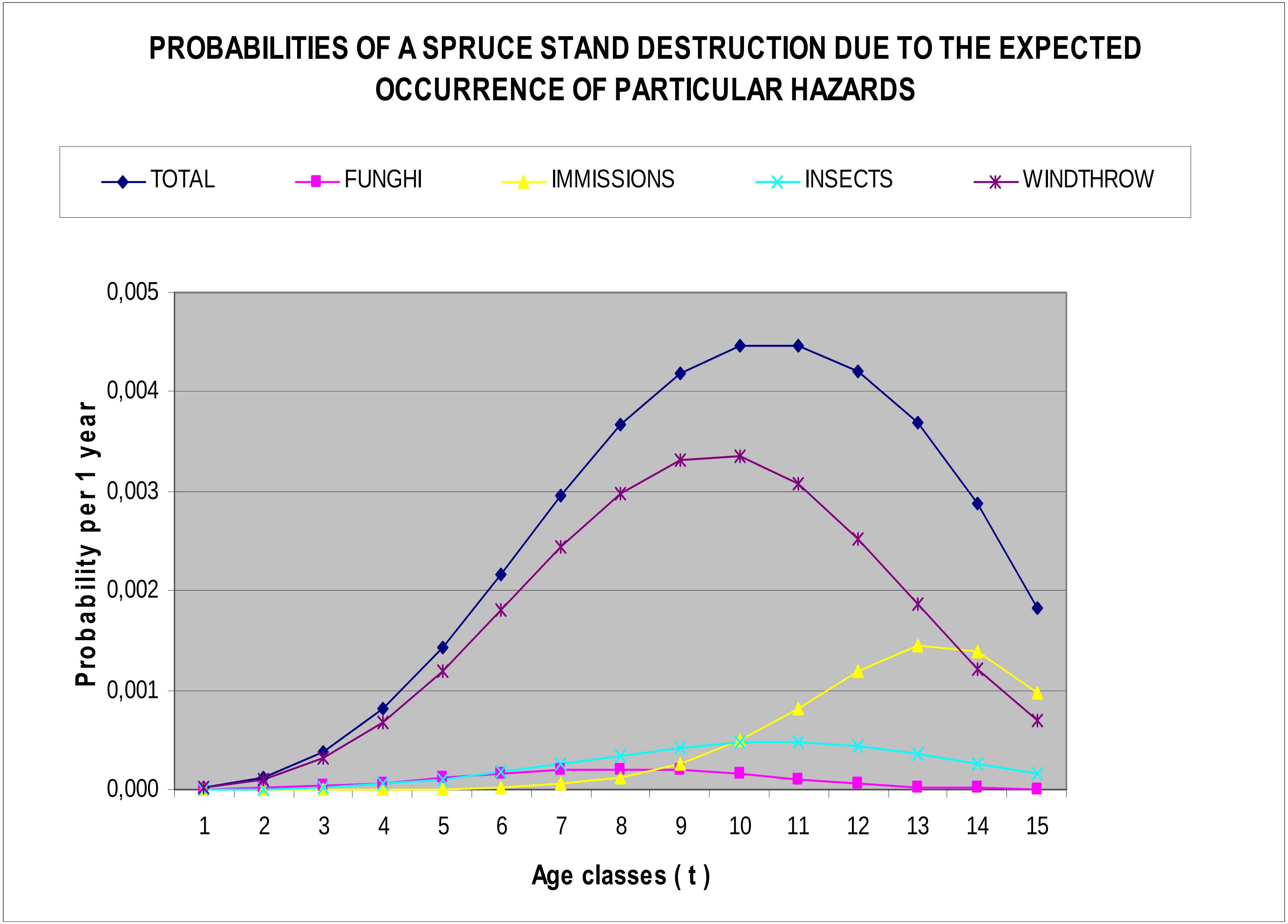
INSECTS AGE (t)	AREA GROWN (ha) A(t)	AREA DESTROYED (ha) a3(t)	POPULATION PROPORTION f(t)	UNIFORM AREA (ha) N(t)	DESTROYED EXPECTED (ha) n3(t)
10	100 564	6,8600	0,000068	96 184,29	6,5612
20	142 236	7,9865	0,000056	96 184,29	5,4007
30	120 596	12,9459	0,000107	96 184,29	10,3253
40	101 685	10,0885	0,000099	96 184,29	9,5428
50	92 353	23,4148	0,000254	96 184,29	24,3862
60	140 941	25,4116	0,000180	96 184,29	17,3420
70	156 814	56,7233	0,000362	96 184,29	34,7922
80	158 220	72,5926	0,000459	96 184,29	44,1302
90	158 583	69,2337	0,000437	96 184,29	41,9917
100	107 606	54,7123	0,000508	96 184,29	48,9050
110	54 157	20,8738	0,000385	96 184,29	37,0722
120	31 036	13,7866	0,000444	96 184,29	42,7261
130	15 429	2,5672	0,000166	96 184,29	16,0038
140	9 292	1,2245	0,000132	96 184,29	12,6746
150	53 251	1,9965	0,000037	96 184,29	3,6061
TOTAL	1 442 764	380,4177	0,003696	1 442 764,30	355,4600

f = 0,000246374

WIND-THROW AGE (t)	AREA GROWN (ha) A(t)	AREA DESTROYED (ha) a4(t)	POPULATION PROPORTION f(t)	UNIFORM AREA (ha) N(t)	DESTROYED EXPECTED (ha) n4(t)
10	100 564	2,2900	0,000023	96 184,29	2,1903
20	142 236	95,8376	0,000674	96 184,29	64,8083
30	120 596	83,4329	0,000692	96 184,29	66,5439
40	101 685	77,7304	0,000764	96 184,29	73,5254
50	92 353	129,3974	0,001401	96 184,29	134,7659
60	140 941	290,8510	0,002064	96 184,29	198,4891
70	156 814	325,8261	0,002078	96 184,29	199,8507
80	158 220	346,5683	0,002190	96 184,29	210,6845
90	158 583	396,2250	0,002499	96 184,29	240,3191
100	107 606	311,8958	0,002899	96 184,29	278,7908
110	54 157	159,5729	0,002946	96 184,29	283,4042
120	31 036	80,8229	0,002604	96 184,29	250,4775
130	15 429	44,1908	0,002864	96 184,29	275,4817
140	9 292	15,0001	0,001614	96 184,29	155,2637
150	53 251	43,1177	0,000810	96 184,29	77,8807
TOTAL	1 442 764	2402,7589	0,026121	1 442 764,30	2 512,4759

f = 0,001741432

SPRUCE AGE	AGGREGATED DESTRUCTION PROBABILITIES	PROBABILITY OF FUNGHI OCCURRENCE	PROBABILITY OF IMMISSIONS OCCURRENCE	PROBABILITY OF INSECTS OCCURRENCE	PROBABILITY OF WINDTHROW OCCURRENCE
(t)	p(t)	p1(t)	p2(t)	p3(t)	p4(t)
10	0,000011879	0,000001163	0,000000000	0,000000502	0,000010214
20	0,000113558	0,000010522	0,000000010	0,000006072	0,000096955
30	0,000370635	0,000032916	0,000000171	0,000022941	0,000314608
40	0,000808711	0,000068707	0,000001246	0,000055686	0,000683072
50	0,001421731	0,000114094	0,000005653	0,000106950	0,001195034
60	0,002164507	0,000160478	0,000019103	0,000176368	0,001808558
70	0,002950539	0,000195740	0,000052731	0,000259183	0,002442885
80	0,003664761	0,000208316	0,000125167	0,000345325	0,002985953
90	0,004194035	0,000192878	0,000262608	0,000419983	0,003318566
100	0,004464889	0,000154100	0,000492131	0,000466507	0,003352151
110	0,004462809	0,000105013	0,000820009	0,000471618	0,003066168
120	0,004206403	0,000060196	0,001190383	0,000431189	0,002524635
130	0,003688119	0,000028575	0,001448772	0,000353507	0,001857265
140	0,002870870	0,000011045	0,001392409	0,000257245	0,001210171
150	0,001829816	0,000003414	0,000970434	0,000164237	0,000691731


PROBABILITIES OF A SPRUCE STAND DESTRUCTION DUE TO THE EXPECTED OCCURRENCE OF PARTICULAR HAZARDS
TOTAL
FUNGHI
IMMISSIONS
INSECTS
WINDTHROW
Probability per 1 year
0,005
0,004
0,003
0,002
0,001
0,000
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Age classes (t)

THE ECONOMIC ANALYSIS OF RISK CONCERNING THE NATURAL ELEMENTS OCCURRENCE

Inputs:

- **NPV(u)** NET PRESENT VALUE of projects concerning spruce growing in (€.ha^{-1}).
- **SV(t)** SALVAGE VALUE of a (t) years old forest stand after its destruction by a natural element in (€.ha^{-1}).
- **SEV(u)** RISK-ADJUSTED SOIL EXPECTATION VALUE at spruce growing in (€.ha^{-1}).
- **SEV$_f$(u)** RISK-FREE SOIL EXPECTATION VALUE at growing spruce in (€.ha^{-1})

1. Net present value of a forestry project ***NPV(u)***

$$NPV(u) = \sum_{t=0}^{u} \frac{R_t - C_t}{(1+r)^t}$$

2. Risk adjusted soil expectation value ***SEV(u)***

$$SEV(u) = \frac{NPV(u) \,.\, (1+\mathrm{r})^{\mathrm{u}}}{(1+\mathrm{r})^{\mathrm{u}} - 1}$$

3. Salvage value of a (t) years old forest stand ***SV(t)***

4. Risk-free soil expectation value ***$SEV_f(u)$*** (REED, 1984) but presented only in the form of a differential equation.

The algorithm of its calculation uses the transition probability matrix ***(W)*** as a tool for the description of age class structure dynamics in the presence of risk.

Transition probability matrix (*W*)

$$
W = \begin{vmatrix}
w_1 & 1-w_1 & 0 & 0 & \ldots & 0 \\
w_2 & 0 & 1-w_2 & 0 & \ldots & 0 \\
. & . & . & . & & \\
. & . & . & . & & \\
. & . & . & . & & \\
w_j & 0 & 0 & 1-w_j & \ldots & 0 \\
. & . & . & . & & \\
. & . & . & . & & \\
. & . & . & . & & \\
w_k & 0 & 0 & 0 & \ldots & 1-w_k
\end{vmatrix}
$$

(w_j) is the probability of a stand destruction in the age class (j)

(w_k) = 1.0

The share of particular age classes (j) on 1 ha originally planted during the assumed decades (i) of a forestry project is given by the elements of vector $\mathbf{p}^{(i)}$:

$$\mathbf{p}^{(i)} = \mathbf{p}^{(0)} \cdot \mathbf{W}^{i}$$

where

$\mathbf{p}^{(0)}$ = [1, 0, 0,…, 0]

KOUBA (1989)

The shares of expected destroyed areas are described by the elements of ($\mathbf{q_{ij}}$):

$$q_{ij} = p_{ij} \,.\, g_j$$

$$g_{j+1} = \prod_{i=1}^{j} (1 - w_j).w_{j+1} \quad \text{pre} \quad g_1 = w_1$$

Gentan probabilities (g_j) were proposed by SUZUKI (1983)

Algorithm of computing the $SEV_f(u)$:

1. Shares of areas **(p_{ij})** and **(q_{ij})** are valuated by particular revenues and costs of a project.

2. Calculating the ***NPV(u)*** of a never-ending forest management project.

3. Algorithm ends when the increment ***Δ NPV(u)*** for the last assumed rotation period of (u) years is less than € 0,01. (HOLECY, 2005)

MEASURING THE ECONOMIC IMPACT OF NATURAL ELEMENTS OCCURRENCE RISK

Output:

- ***RPSEV(u)*** RISK PREMIUM ON THE FOREST SOIL management

$$RPSEV(u) = SEV(u) - SEV_f(u)$$

$$RPSEV(u) = 2\,311\ € \,.\, ha^{-1} - 1\,683\ € \,.\, ha^{-1}$$

$$\mathbf{RPSEV(u) = 628\ € \,.\, ha^{-1}}$$

TABLE 1: Input data for the spruce management project

SPRUCE:			yield-class:	28
AGE (t)	STUMPAGE VALUE ST(t)	THINNINGS VALUE TH(t)	REVENUES R(t)	COSTS C(t)
(years)	(€ .ha^{-1})	(€ .ha^{-1})	(€ .ha^{-1})	(€ .ha^{-1})
0	0	0	0	1648
10	0	0	0	1252
20	0	0	0	287
30	911	140	140	104
40	2028	299	299	104
50	3548	451	451	104
60	5023	535	535	104
70	5895	553	553	104
80	6989	551	551	104
90	7664	524	524	104
100	8550	535	535	104
110	9220	504	504	104
120	10488	521	521	104
130	10982	501	501	104
140	11598	471	471	104
150	12085	437	12522	104

TABLE 2: The economic analysis of a spruce management project according to the expected decennial cash-flows

SPRUCE:	yield-class: 28						
AGE (t)	DISCOUNT FACTORS (p = 0.01 p.a.)	DISCOUNTED STUMPAGE DST(t)	DISCOUNTED THINNINGS DTH(t)	DISCOUNTED THINNINGS ACCUMMULATED DTHA(t)	DISCOUNTED COSTS ACCUMMULATED DCA(t)	NET PRESENT VALUE NPV(u)	SOIL EXPECTATION VALUE SEV(u)
(years)		(€ .ha^{-1})	(€ .ha^{-1})	(€ .ha^{-1})	(€ .ha^{-1})	(€ .ha^{-1})	(€ .ha^{-1})
0	1,000000	0	0	0	1648	-1648	
10	0,951466	0	0	0	2840	-2840	-58507
20	0,861349	0	0	0	3087	-3087	-22263
30	0,779768	711	109	109	3168	-2348	-10659
40	0,705914	1432	211	320	3241	-1489	-5062
50	0,639055	2267	288	608	3307	-431	-1195
60	0,578528	2906	310	918	3367	457	1085
70	0,523734	3087	289	1207	3421	874	1835
80	0,474129	3314	261	1469	3470	1312	2496
90	0,429223	3290	225	1694	3514	1469	2574
100	0,388570	3322	208	1902	3555	1669	**2730**
110	0,351768	3243	177	2079	3591	1731	2671
120	0,318451	3340	166	2245	3624	**1960**	2876
130	0,288289	3166	144	2389	3654	1901	2671
140	0,260984	3027	123	2512	3681	1858	2514
150	0,236266	2855	103	2615	3705	1765	2311

TABLE 3: The calculation of the Vulnerable Forest Stand Value according to its age

SPRUCE:	yield-class:		28	
AGE (t)	FOREST EXPECTATION VALUE FEV(t)	SALVAGE VALUE OF FOREST STAND SV(t)	RISK PREMIUM ON FOREST SOIL RPSEV(u)	VULNERABLE FOREST STAND VALUE VFSV(t)
(years)	(€ .ha^{-1})	(€ .ha^{-1})	(€ .ha^{-1})	(€ .ha^{-1})
0	0	0	628	628
10	1850	0	628	2479
20	3669	0	628	4297
30	4611	0	628	5240
40	5295	0	628	5924
50	5875	1821	628	4683
60	6348	2282	628	4695
70	6777	2646	628	4760
80	7232	2943	628	4918
90	7736	3207	628	5158
100	8323	4684	628	4267
110	8958	4878	628	4709
120	9695	5006	628	5318
130	10490	5159	628	5959
140	11391	5327	628	6692
150	12419	5421	628	7626

CONCLUSIONS

1. **The risk is considerably higher at growing older spruce stands.**

2. **The impact of the all disturbances on the capital value of forest land is very strong.**

3. **The risk of management at growing pure spruce stands could be decreased by planting mixed forests composed also of beech, maple and ash-tree.**

INSTITUTIONAL FRAMEWORK FOR ACCEPTING THE RISK IN FORESTRY

Negative impacts of risk attending the forest management:

- Deterioration of forest and other ecosystems
- Decreasing the quality of the environment
- Disturbing the sustainability of forestry
- Reduction of social benefits accruing from forestry
- Decreasing the profitability of forest enterprises
- Decreasing the financial stability of forest enterprises

Remedial interventions to cope the presence of risk in forestry:

- Ecosystem management approach.
- Planting the ecologically more stable mixed forests.
- Preservation and strengthening the biological diversity.
- Collaborative and participative planning approaches.
- Insurance of spruce forests in order to strengthen the financial stability of forest land management.

THANK YOU FOR YOUR ATTENTION !